microstrip parameters ϵ_{eff}, Z_c, α_c, and α_d for any provided w and f.

The two-dimensional interpolation for this problem has been set to include cubic spline functions of f, and parabolic spline functions of w. For the w-dependence the interpolation is carried out *between* knots while for the f-dependence it has been arranged *at* knots. As mentioned in the preceding section, these arrangements raise the accuracy and stability of the algorithm. The orders of the spline functions were experimentally determined for the desired stability, and to give a reasonable compromise between CPU time and accuracy.

Table I displays some of the exact data set for ϵ_{eff} alongside values computed by the bivariate interpolation program. The frequencies and widths were successive 'pseudo-random' numbers generated over the valid ranges 1 to 9 GHz and 0.1 to 1.1 mm, and rounded to 4 decimal figures before use as data for the test. The differences in this Table indicate that the interpolation error is indeed very small, with an rms error of less than 0.04 percent. Through experiments with the program, it was found that the CPU time for computing a set of microstrip parameters is at least 100 times less that of the original program. The final CPU time is then of order tens or hundreds of milliseconds, depending on the range of parameters, the computer, etc.

IV. Conclusions

The lack of accuracy in earlier approximate solutions for the microstrip line gives the motive to find a new, accurate, and fast technique for computation in times appropriate for CAD purposes. A bivariate interpolation has been used to compute all the microstrip parameters with high accuracy and efficiency. The data base is provided in a 'once-for-all' computer run with a very accurate program and analysis. In subsequent computation, perhaps as part of an interactive or automated design procedure, the bivariate interpolation makes use of the basis-spline functions and the tensor product.

Following this same technique, described for microstrip, other planar lines can be similarly programmed. Interpolation by basis-spline functions becomes distinctly superior for lines (or circuits) whose characteristics are not easily approximated on physical, *a priori* grounds.

Although this bivariate interpolation is adequate for many circumstances, circuits with more than two 'continuous variables' (such as strip width, strip separation, and frequency of a microstrip directional coupler) need a more general multivariate algorithm. To the authors' knowledge, no such efficient algorithm has yet been established.

Postscript

Referees have suggested possible publication of the associated computer programs. The two programs have no documentation suitable for publication, but copies of the extant programs (in Fortran IV) may be obtained from J. B. Davies.

References

[1] E. Yamashita, K. Atsuki, and T. Ueda, "An approximate dispersion formula for microstrip lines for computer-aided design of microwave integrated circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 1036–1038, Dec. 1979.

[2] S. Y. Poh, W. C. Chew, and J. A. Kong, "Approximate formula for line capacitance and characteristic impedance of microstrip line," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 135–142, Feb. 1981.

[3] D. Mirshekar-Syahkal and J. B. Davies, "Accurate solution of microstrip and coplanar structures for dispersion and for dielectric and conductor losses," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 694–699, July 1979.

[4] D. Mirshekar-Syahkal and J. B. Davies, "Computation of the shielded and coupled microstrip parameters in suspended and conventional form," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 274–275, Mar. 1980.

[5] C. de Boor, *A Practical Guide to Splines*. New York: Springer-Verlag, 1978.

[6] G. E. Forsythe, M. A. Malcolm, and C. Moler, *Computer Methods for Mathematical Computations*. Englewood Cliffs, NJ: Prentice-Hall, 1977.

[7] J. Stoer and R. Bulirsh, *Introduction to Numerical Analysis*. New York: Springer-Verlag, 1980.

[8] C. de Boor and G. H. Golub, *Recent Advances in Numerical Analysis*. New York: Academic Press, 1978.

A Technique for Measuring the Effective Dielectric Constant of a Microstrip Line

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Abstract —**A method is discussed by which the effective dielectric constant of a transmission line of complex cross section is determined experimentally.**

In an effort to determine the insertion loss represented by a microwave filter, it becomes necessary to determine the effective dielectric constant and the characteristic impedance of an unusual type of microstripline. The microstripline, as seen in Fig. 1, consists of a metal strip mounted on a dielectric slab which, in turn, is suspended over a ground plane.

Since this type of microstrip structure does not appear to be amenable to conventional microstrip formulas, it seems that a combination of theoretical and experimental techniques would be necessary in order to predict the line's parameters. Since the purpose of this letter is to emphasize the experimental techniques, only a brief summary will be given on the theoretical aspects. To theoretically determine the effective dielectric constant and characteristic impedance a variational technique developed by E. Yamashita and R. Mittra [1] is used, along with standard transmission-line-type formulas. To be more specific, once the distributed capacitance is determined for the inhomogeneously layered line ($\epsilon_s \neq 1$) (see Fig. 1), and the homogeneous line ($\epsilon_s = 1$), the ratio of these quantities gives the effective dielectric constant. With the effective dielectric constant known, the characteristic impedance can then be approximated by using conventional transmission-line formulas along with the homogeneous ($\epsilon_s = 1$) capacitance, which is determined by the variational formulation previously mentioned.

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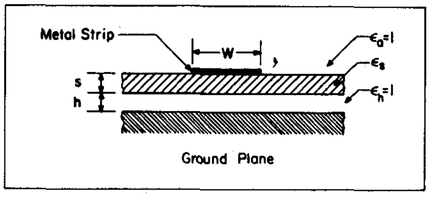

Fig. 1. Suspended-air microstrip configuration and notation.

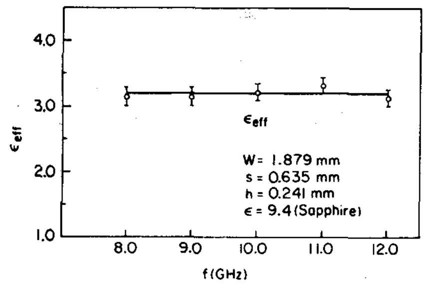

Fig. 2. Effective dielectric constant versus frequency.

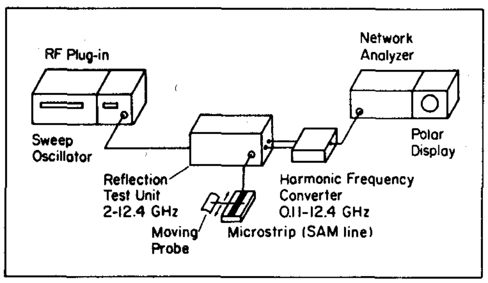

Fig. 3. Experimental setup for measuring effective dielectric constant.

Fig. 2 shows a plot of effective dielectric constant versus frequency. The solid line represents the variationally determined effective dielectric constant under zero dispersion conditions (i.e., the TEM approximation) [2]. The data points shown were determined with the technique to be described.

Fig. 3 shows the experimental setup. The suspended-air microstrip (SAM line) test block is shown next to the adjustable field probe. This setup is used to measure the guide wavelength (and consequently, the effective dielectric constant) in the following manner. By moving the adjustable probe across the top of the microstrip (about 1/16 in above the metal) a corresponding movement on the polar display (HP-8414A) can be seen. By starting at a point on the strip which is not too close to either end, and then moving until the dot on the polar display returned to its approximate original position, we are able to measure a distance corresponding to one-half the guide wavelength. To measure this distance, the moving probe has a calibration scale attached to its mounting, thus ensuring quick and accurate readings. Since we were operating at 10 GHz, the setup involved allowed us to measure two guide wavelengths, thus improving the accuracy of the readings by allowing us to determine the average guide wavelength. If the line is ideal (no losses and no dispersion) the positions of the dot on the polar display would more nearly coincide. Care must be taken to minimize the deleterious effects of the presence of the probe (keeping the probe as far away from the line, observing the probe effect when at a field minimum if not matched, and using any other least-coupling means).

Consequently, by knowing the frequency of the oscillator and the guide wavelength λ_g, it is easy to determine the effective dielectric constant from

$$\epsilon_{\text{eff}} = \left(\frac{\lambda_0}{\lambda_g}\right)^2 \tag{1}$$

where λ_0 is the free-space wavelength.

Given the effective dielectric constant and the capacitance C_0 (variationally determined for $\epsilon_s = 1$) we can then determine the characteristic impedance from

$$z_c = \frac{1}{v_0 C_0 \sqrt{\epsilon_{\text{eff}}}} \tag{2}$$

where v_0 ($= 3 \times 10^8$ m/s) is the velocity of light in the free space. Since frequency dispersion is negligible at this frequency (see Fig. 2), the previous transmission-line formulas are an excellent approximation.

References

[1] E. Yamashita and R. Mittra, "Variational method for the analysis of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 251–256, Apr. 1968.

[2] R. Mittra and T. Itoh, "Charge and potential distributions in shielded striplines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 149–156, Mar. 1970.